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ON THE SCALING PROPERTIES OF A STOCHASTIC RAINFALL MODEL

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A class of stochastic rainfall models known as 'renewable process rainfall model' is strong in representing physical patterns found in the rainfall series. A stochastic rainfall model based on the renewable process was constructed and possible scaling behavior across different time-resolutions were examined. It was discovered that most of the model parameters obey power-law type scaling. The result suggests the possibility of generating synthetic rainfall series of higher resolution from low-resolution observations with these scaling properties.

Key Words: *Rainfall Model, Stochastic Model, Scaling*

1. INTRODUCTION

Rainfall measurement is a time-consuming exercise, which demands a considerable amount of effort and involvement. Two main factors govern the analytical value of rainfall data series, namely the duration of measurement and the temporal intensity. For low resolutions like daily scale, many years of records are available for many geographical locations. However when it comes for high resolutions like hourly or less the availability in terms of location as well as duration becomes very much limited. For sub-hourly time-scales like 10min or 1min, the records are limited to a few experimental meteorological stations. The recent shift of focus to sustainability and environmental concerns has triggered a demand of rainfall data at much higher resolutions than generally available. There are many occasions where the problem at hand require a different time-resolution than that of the available rainfall data. Due to the irregular nature of rainfall series, this relationship between different resolutions is not a trivial one. Thus, the problem of relating the rainfall records of various resolutions has become a timely topic.

Most of the 'traditional' stochastic methods employed to analyze hydrological time-series including rainfall, focus only on a fixed time-scale. There are many examples of rainfall models, which work at hourly or larger scale. The problem of relating the stochastic rainfall model behavior at various time-scales is yet to be addressed. On the other hand fractal and multifractal scaling properties of rainfall time-series have been extensively studied in the last decade (Tessier, *et al.* 1996[8)], Olsson 1996[7)], de Lima 1999[5)]) . Though most of these efforts have mainly concentrated on the analysis of rainfall properties than using it for actually predictable rainfall models, they have revealed some interesting scaling relationships across various time resolutions. One of the common features of most of these fractal models is that they address the analysis of rainfall series in the typical time-series fashion. The occurrence/non-occurrence of rainfall can be analyzed using single fractal theory, which involves 'box counting', or finding the number of cells of a given size which contains rainfall – along the time dimension. The incorporation of rainfall quantity dimension to this problem, namely considering the intensity, is done generally by multifractal measures. Multifractal methods that are popular for rainfall analysis, mostly use some modified method of box counting that incorporate the intensity(Olsson and Niemczynowicz 1996[6)]). The most difficult question one is faced, during the process of us-

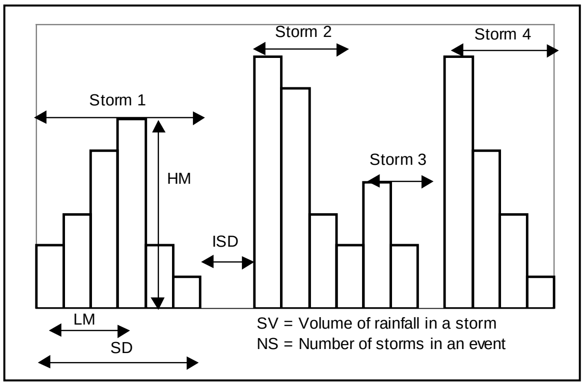

Fig. 1 Six parameters used to describe the structure of a rain event.

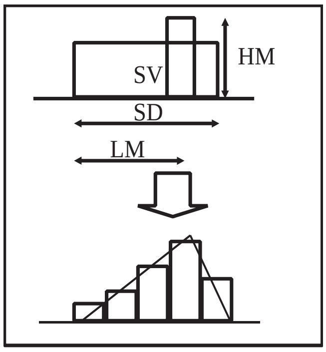

Fig. 2 Constructing a rain storm using the model output.

ing these models to generate synthetic rainfall series, is that they are based mainly on 'collection' statistics and do not pay much attention of the structure of the rainfall series.

One can find a family of rainfall models which gives primary attention to the physical shapes of the rainfall-series in the recent literature(Croley, et.al. 1974[1)], Ethoh and Murota 1986[2)], Lardet and Obled 1994[4)] and Haberlandt 1998[3)]). Rather than modeling the rainfall process as a succession of pulses of various intensities, these consider some 'collective' measures of a number of rain pulses next to each other, which defines a physical shape. In the present paper we model rainfall at different resolutions using a 'renewal process' (Lardet and Obled 1994[4)]) rainfall model. Model parameters are then investigated for possible scaling relationships.

2. THEORY

A Rainfall series can be considered as a succession of alternative dry and wet spells, whose designation depends on the rules defined in the context of the particular application problem. Most important rule is the length of the minimum dry period, which separates one rain event from the other. Generally the nature of the problem at hand has much to do with this definition. For example if the interest is the runoff prediction for flood problems, then a shorter dry period may be considered than that of a ground water related problem. One of the key assumptions of the present model is that the one rain event is independent of the other rain events in the rain series. Thus, a duration, which is large enough to have small autocorrelation, should be used.

Figure 1 shows the key parameters of the model in defining a rainfall event. The rainfall event is modeled as a collection of rain storms of triangular shape, a fact largely verified by the observations (Lardet and Obled 1994[4)]). A storm as defined in this model has a single peak value. The properties of the whole event like the duration of the rainfall event and the rainfall volume involved are not directly used in the model. The properties of a rainfall event are defined using the number of storms in that event and the statistical properties of the storms. This leads to another key assumption that storms that constitute the event are independent of each other.

The external structure of the model is defined by four parameters, namely the number of storms in the rainfall event (NS),the length of the dry period between two storms (ISD) and the duration(SD) and the volume(SV) of each storm. The internal structure of a storm is defined by two parameters: the peak intensity (HM) and the time duration from the start of the storm to the occurrence of the peak (LM). The four external parameters together with two internal parameters fully define the rainfall event.

It is evident both from the general perception of the problem as well as examining rainfall data that some of these parameters have strong correlations. Secondary parameters were defined to reduce these dependencies. Once the distributions for six parameters, which are independent of each other, are obtained from the observations, it is possible to generate rainfall series synthetically by generating values for those parameters sequentially, and then compute the storms using the triangular distribution (Figure 2)

3. RAINFALL DATA

A rainfall record of four years from a rain gauge situated in Maehara Catchment in Chiba prefec-

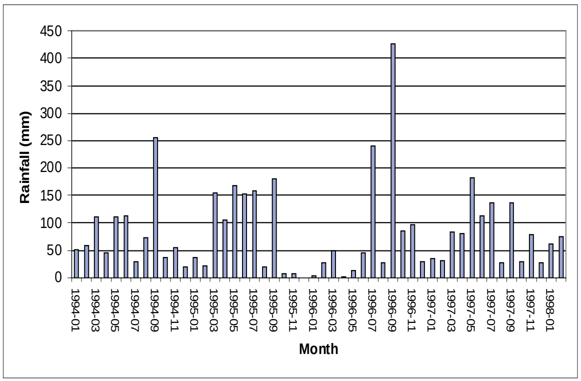

Fig. 3 Monthly average rainfall values of the available rainfall data.

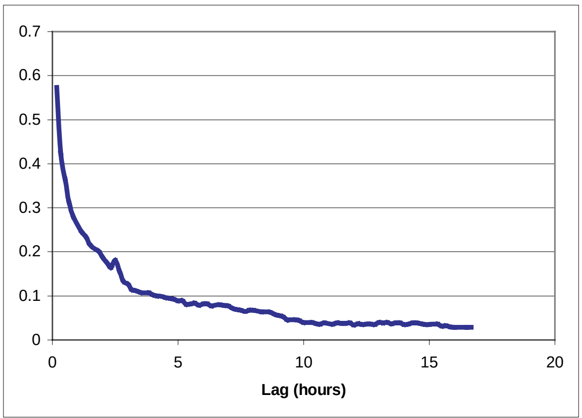

Fig. 4 Autocorrelations of the rainfall data.

ture, by the water resource-engineering laboratory of the University of Tokyo, was used for the analysis. The rain gauge was tipping bucket type with the tip size of 0.5mm. The gauge automatically logs the number of tips at every ten-minute interval, providing a time-series of 10min resolution. In order to ensure temporal homogeneity, only the duration from the beginning of May to end of July was considered for the analysis. This covers a time-period which is regularly 'rainy' (figure 3.), but avoids the rains dominated by typoons.

4. APPLICATION

Six hours was taken as the minimum stretch of dry spell which can separate to rain events. Autocorrelation of the series (See Figure 4, the value for 6hrs is less than 0.1) show that the assumption that events are independent of each other, holds well at this level of separation. Each of the six variables was calculated and the

Table 1 Correlation among the parameters.

	SD	ISD	SV	LM	HM
SD	1	-0.285	0.533	0.760	0.415
ISD		1	-0.187	-0.167	-0.119
SV			1	0.466	0.853
LM				1	0.315
HM					1

cross-correlations were examined. Table 1 shows the correlation matrix for the resolution of one hour. It was identified that the pairs (SD,SV), (SV,HM), (SD,LM) have strong correlations and hence cannot be generated one followed by the other. The values of these high correlations seem to be quite similar to those obtained in a previous study of hourly rainfall records in Japan(Etoh and Murota 1986[2)]). Further it was observed that these strong correlations are prevailing at all the scales studied. Auxiliary variables were defined to handle these dependencies, as follows:

$$SV = SD^{RSVSD} * SVSD \tag{1}$$

$$HM = SV^{RHMSV} * HMSV \tag{2}$$

$$LM = SD^{RLMSD} * LMSD \tag{3}$$

The model structure was defined in such a way that, (for example in the case of SV, SD) first SVSD is generated and then using the distributions of SVSD and SD and the value estimated for RSVSD, the value for SV is computed by (1). By this method the statistical relationship between SV and SD are maintained. Probability distributions were fitted for each of the parameters. Most of the parameters fitted well with either exponential or log-normal distribution (See Figures 5 and 6 for examples).

(1) Model Verification

In order to test whether the overall statistical properties of the observations are adequately represented in the model, the statistics, which are not involved in the modeling process, were compared. The event duration (ED) and the total rainfall volume in a rainfall event (EV) are such parameters. The comparison of the distribution of these two parameters are shown in figures 7 and 8.

(2) Scaling Properties

It was observed that the power-factors (RSVSD,RLMSD and RHMSV) have very close values at different resolutions. Figure 9 shows the

Fig. 5 Probability distribution for SVSD.

Fig. 6 Probability distribution for SVSD.

values estimated for these factors by linear regression at various time-steps. The average value of each of these factors was used when comparing the model parameters. The probability distributions used to model the model parameters, namely exponential and log-normal distributions, are defined fully with the arithmetic mean of the distribution and mean and variance on logarithmic scale respectively. Hence, the mean and variance of distributions for each coefficient at different time steps were examined. Some of the scaling relationships are illustrated in Figures 10,11 and 12. Figure 10 shows the scaling of the variables NS and SD. NS varies in a linear fashion,

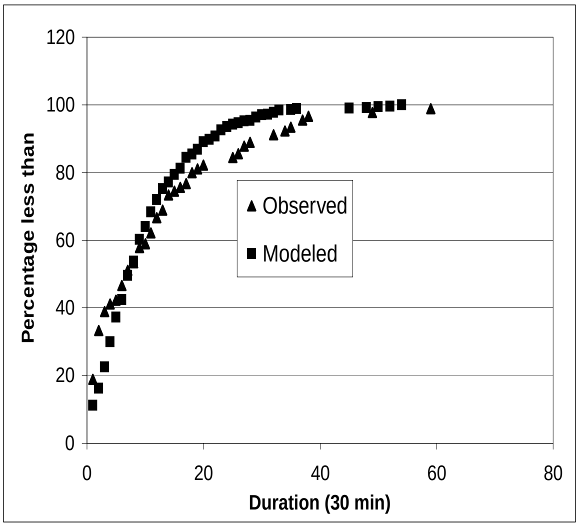

Fig. 7 Event duration distributions at 30min resolution.

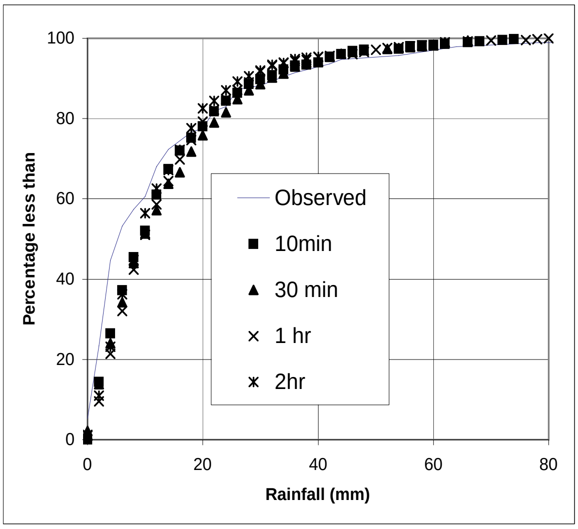

Fig. 8 Event volume distributions.

whereas the variation of SD is close to a hyperbolic function. This further indicates the (intuitive) complimentary relationship between the two variables. Almost all of the parameters have their first moments scaling according to a power law.

(3) Predictability

In order to examine the quality of possible generation of higher resolution rainfall from those of lower resolution, the following exercise was carried out. Using the parameter estimates for 30min and lower resolutions and assuming a power-law scaling in general, predictions were ob-

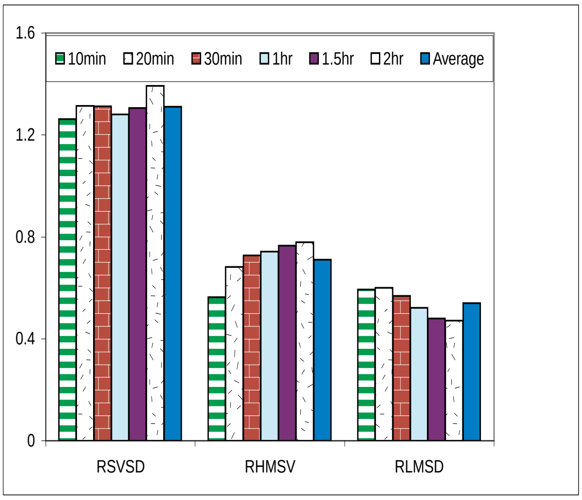

Fig. 9 Values of power-factors at various resolutions.

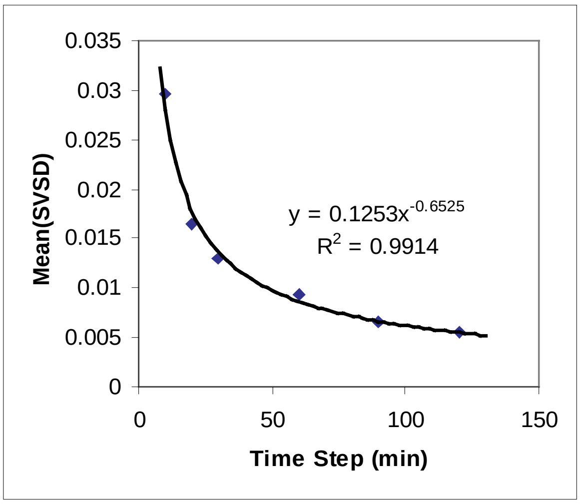

Fig. 11 Scaling of the mean of SVSD.

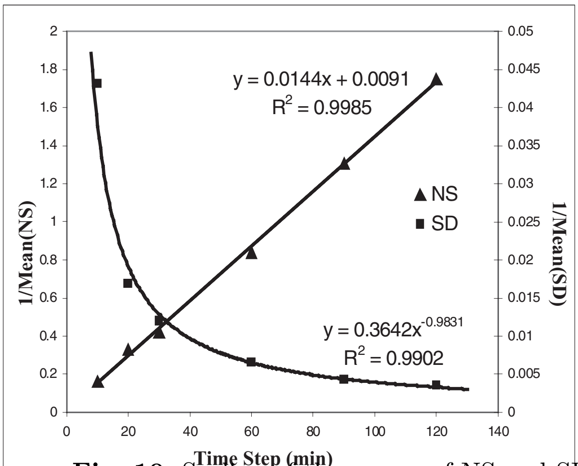

Fig. 10 Scaling of the means of NS and SD.

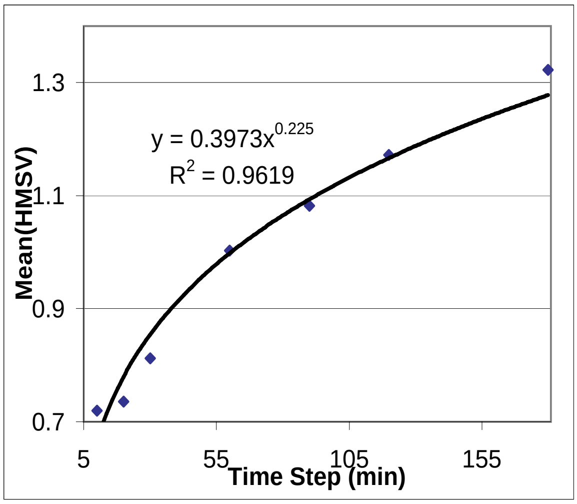

Fig. 12 Scaling of the mean of HMSV.

tained for the parameters for 10min and 20min resolutions. Using these extrapolated parameters, synthetic rainfall series was developed. It was observed that the statistical properties of generated 10min and 20min series using these extrapolated values agree well with the series generated from the parameters estimated from the original 10min and 20min series. Figure 13 shows the comparison of the distribution of event volume (EV) at 10min resolution.

5. CONCLUSIONS AND DISCUSSION

A 'renewal process rainfall model' was used to model the rainfall observed in Maehara, Chiba Prefecture in Japan. The statistics of the rainfall generated from the model agree fairly well with those of observed rainfall. It was found that the same model structure could be used to model the rainfall series at different resolutions. The distributions of parameter values of the model for same rainfall series at different time resolutions show well defined scaling properties following power law in general.

The model behavior in the resolutions ranging from a few hours to ten minutes were examined in the present study. The rainfall structure at this range of resolutions seems to be modeled reasonably well with the current approach, involving triangular shape based storm. However, the applicability of the model to coarser resolutions, (like daily scale) might not be equally successful

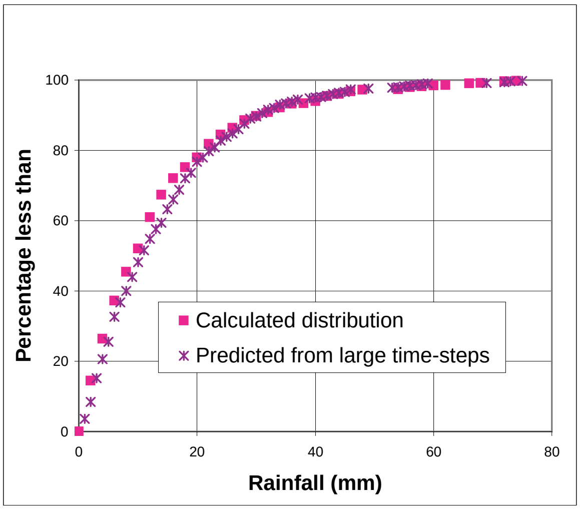

Fig. 13 Comparison of cumulative event volume distribution at 10min resolution.

due to the differences in the structure of the rainfall. Such extensions of the applications should be carefully verified.

The practical use of the approach would depend on how closely the rainfall series can be modeled with the 'renewal process' rainfall model. Once it is modeled at a few coarse resolutions, higher resolution model parameters can be estimated assuming power law type distributions to model at higher resolutions. Further work is required in this direction.

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REFERENCES

1) T. Croley, R. Eli, and J. Cryer. Ralston creek hourly precipitation model. *Water Resources Research*, 14(3):485–490, 1978.
2) T. Etoh and A. Murota. Probabilistic model for rainfall of a single storm. *Journal of Hydroscience and Hydraulic Engineering*, 4(1):65–77, 1986.
3) U. Haberlandt. Stochastic rainfall synthesis using regionalized model parameters. *Journal of Hydrologic Engineering*, 3(3):160–168, 1998.
4) P. Lardet and C. Obled. Real-time flood forecasting using a stochastic rainfall generator. *Journal of Hydrology*, 14(162):391–408, 1994.
5) J. Gasman M. I. P. de Lima. Multifractal analysis of 15-min and daily rainfall from a semi-arid region in portugal. *Journal of Hydrology*, 220:1–11, 1999.
6) J. Olson and J. Niemczynowicz. On the possible use of fractal theory in rainfall applications. *Journal of Geophysical Research*, 101(D21):26,427–26,440, 1996.
7) J. Olsson. Validity and applicability of a scale-independent multifractal relationship for rainfall. *Atmospheric Research*, 42:53–65, 1996.
8) Yves Tessier, Shuan Lovejoy, Pierre Hubert, and Sean Pecknold Daniel Schertzer. Multifractal analysis and modeling of rainfall and river flows and scaling, causal transfer functions. *Journal of Geophysical Research*, 101(D21):26,427–26,440, 1996.