FRACTAL MODELING OF RAINFALL:
DOWNSCALING IN TIME AND SPACE FOR HYDROLOGICAL APPLICATIONS

雨のフラクタルモデリング:
水文学的応用に向けた時空間ダウンスケーリング

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There is an urgent need of high-resolution meteorological information to cater for the new focus on the hydrological response at smaller spatial and temporal scales, which has arisen largely due to the problems related to urbanization. Alternative means of estimating precipitation at small scales are beneficial, as high-resolution data acquisition is a time consuming and expensive task. Successful spatial and temporal downscaling methods find many uses in the process of solving practical hydrological problems, due to the fact that they can benefit from the availability of extensive historical records at lower resolutions. These include deriving of high resolution synthetic data, based on low resolution observations; interpolation of the measurements of low density rain gauge networks; combining of data sources of different resolutions and reliability, like gauge based rainfall and weather satellite data; and downscaling of the output of regional climatic models and global circulation models.

In this report, a series of attempts to downscale rainfall fields in space and time using the mathematical theory of fractal scaling is presented. First, a comprehensive review on the recent findings on fractal scaling and its applications on modeling rainfall was done, in order to understand the state of the art of this approach. Last two decades have seen the introduction of fractal and multifractal theories and a considerable amount of theoretical work, including a number of mathematical models on multiscaling. A number of applications on precipitation, most involving rainfall data of various locations of Europe, North America and Oceania, had concluded that rainfall shows fractal scaling properties. There are good reasons for performing this type of analysis in Asia: The nature of the rainfall of Asia is distinctly different from many locations where multifractal studies had been already reported, mainly due to monsoon effect and occurrence of cyclones. The phenomenally high urbanization rate of the region makes it increasingly vulnerable for flash floods. Compared with countries of Europe, North America and Oceania, those of Asia have a severely limited amount of good quality precipitation data at high temporal and spatial intensities. Japan is the special case in Asia, where extensive databases of high-resolution precipitation is available. The investigation of Japanese rainfall is an excellent opportunity to gain an insight into the fractal features of the Asian Rainfall.

The main objective of this study was to apply the already available knowledge of multiscaling of rainfall to device means to solve precipitation related problems in operational hydrology, that involves scaling. However, the questions: whether the precipitation

is in fact multiscaling and whether the multifractal theory can adequately capture the variability in rainfall in time and space, had to be addressed in order to model the scaling properties accurately. Where possible, the limits of the multifractal scaling regimes were inspected. Most of the analyses and model validations were performed on precipitation data of Japan.

In order to investigate the existence and nature of possible fractal scaling in rainfall, a number of studies were performed. These were the first investigations reported on the multifractal scaling of Japanese rainfall. First, the scaling in time was examined using a large number of gauge observed time-series from all over the Japan. The scaling regime of the investigated data extending from hourly scale to daily scale, found to be breaking at a scale around two days. This upper-break was less than those reported by many other studies hitherto. The spectral slopes where much closer to unity and were somewhat larger than similar past studies.

The specific problem of deriving hourly synthetic rainfall series from observations made at daily scale was attempted based on the results of multifractal analysis. The apparent break of the scaling regime subsequent to two days made it difficult to use the conventional modeling strategies that require a continuous scaling that spans over at least several different scales. A new modeling approach based primarily on the geometrical properties of characteristic function of multifractal scaling, that can work satisfactorily with the small range of available scales of daily and two-day resolutions, was proposed to overcome the problem. Validation of hourly synthetic rainfall data derived from this method, by statistically comparing that data with hourly-observed series showed that the synthetic data closely resembles observations.

Due to the limitations of resolution and precision of commonly available rainfall data, scaling behavior of rainfall below the hourly scale had not been widely studied. In order to examine the existence and the nature of scaling below hourly scale, several high-resolution rainfall series were obtained by performing a rainfall measuring experiment in several locations in *Maehara*, Chiba prefecture. The results of analysis of data of a period of one year indicated that the scaling properties are extending below hourly scale, at least down to 5min resolution. This result implies that, at least in principal, it is possible to relate a scale as small as 5min to daily rainfall observations.

Spatial scaling of Japanese rainfall data was investigated using gauge interpolated rainfall maps. Before the analysis, the network of rain gauges was examined to evaluate the homogeneity of the distributions and the resolution limits of the analysis, to minimize the effect of the artificial smoothness introduced by interpolation, on the scaling properties derived. Scales between 0.1^0 and 0.8^0 were subjected to analysis. The original time-integration of the rainfall snapshots was one hour. As expected, the analysis indicated a higher variability/intermittency of spatial rainfall than the previously reported studies involving daily accumulations. A secondary analysis with daily accumulations produced multifractal model parameters that are comparable with past studies.

Multifractal properties of spatial rainfall showed some important relationships with the other rainfall related parameters. One is the strong dependency of the former with the large-scale forcing, or the grid-averaged rainfall. The multifractal parameters demonstrated a strong seasonal behavior indicating a sharp anomaly around August, too. This anomaly could be explained by the frequency of occurrence of rainfall of different types in each season.

Radar based measurement of rainfall has the advantage of being a truly spatial measure, compared with gauge based data. Further, it is possible to investigate the rainfall process at much higher spatial resolutions using radar information. However, these estimations are not direct measures rainfall quantity. Therefore, radar estimated rainfall may not be comparable with gauge measurements in terms of quantitative accuracy. A a multifractal model was used to perform a comparative analysis of the two sources in terms of spatial variability. While, the two sources produced closely matching results, there was a discrepancy in scaling properties that could be explained by the differences in extreme values.

The rainfall over a large area generally shows some regular spatial heterogeneity, due to many reasons that include orography and slope aspects. When the scale of temporal accumulation is small, this heterogeneity is hidden in the high variability or the randomness of the process. However, at larger accumulation lengths, rainfall shows distinct patterns that reveal the spatial heterogeneity, for the randomness becomes lesser and lesser with increasing integration volumes. Since multifractals are processes that are statistically homogeneous in space, pure multiscaling fields cannot represent this spatial heterogeneity, which is an inherent characteristic of the aerial precipitation. A new multifractal-based spatial disaggregation model, which can accommodate the spatial patterns at long accumulation sizes, was proposed. The model has two components: A deterministic part that maintains the spatial heterogeneity apparent in large accumulations and a multifractal model to describe the randomness that is predominant in short integration sizes. This separation of rainfall generation into two phases is consistent with the empirical understanding of rainfall over land. The combination of the two components result in a process, which can produce rainfall fields that has both the properties of the small-scale variability and the long-term spatial patterns. Since the heterogeneity and the random variability are independently considered in the model, it has the unique advantage of the ability to use any multifractal model for modeling the latter. This fact was demonstrated by using two multifractal models.

Application of the model on radar based daily rainfall over central Japan, produced encouraging results. The disaggregation based on the large-scale forcing could maintain the observed long-term heterogeneity of the model area accurately. The model was validated by comparing the rainfall intensity distributions at a number of points having small, moderate and large rainfalls. The reasons for this model to work well for autumn, summer and spring, but to fail in winter, were explained.

Extensive analytical studies showed that the rainfall in Japan show multiscaling properties. A concrete method to downscale daily rainfall into 7 hourly scale was proposed and validated. A new spatial rainfall model, which can disaggregate rainfall maintaining the long-term spatial heterogeneity, was implemented. This could create daily rainfall distributions with properties similar to observations. To conclude, this study revealed the potential of the multifractal scaling in solving practical hydrological problem of downscaling rainfall in space and time.

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Assela Pathirana
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Chapter 1

Introduction

There a two streams of geophysical engineering that are developing at a remarkable phase from the beginning of the last decade, namely the study of the earth's atmosphere and related phenomena from a holistic point of view and the development of hydrological analysis and modeling techniques that are focused on increasingly smaller regions. Largely, both of these development trends are results of attempts to understand and counter the human made changes that have adverse impacts on the environment and threatens the safety and sustainability of the human society.

The concerns about the global warming and its consequences like climatic change have triggered an increased interest in study on global or regional climate and atmospheric phenomena. Recent years have seen a tremendous amount of research to improve the understanding of the mechanisms of the atmosphere that regulate the temperature, winds and precipitation. In addition to the intended scientific objectives, this environment has provided excellent breeding grounds for a host of improved climatic models that attempt to forecast the wind and precipitation patterns at regional scales.

Rapid urbanization has created many hypersensitive localities that respond to the forces of the hydrological cycle in a remarkably unorthodox manner. A prominent characteristic of the urban hydrological systems is the swiftness of change: both in time and space. Last few decades have seen an increasingly high number of a new type of floods that occur within hours of high rainfall. These flash floods are partly the results of the increased clearing of watersheds of vegetation and decrease of the infiltration capacity of the soil surface, mainly due to construction activities. On the other hand, the so called flash floods have been increasingly noticed, due to the stakes, both in terms of financial value and human life, that are concentrated on small extents of vulnerable land on urban environment. Regardless of the reasons, urban flash floods have become vicious problems that have to be mitigated by the hydrological engineers. The major challenge that needs to be addressed in the fact that the scales involved in urban hydrology are much smaller than those of conventional hydrological studies. Many hydrological variables have to be quantified at these small space time scales. Among these variables, precipitation is the most important, due to its being the driving force of the hydrological cycle, urban or otherwise.

Mathematically, fields of values in space or time can be classified in to two broad categories based on their behavior at different scales[1]. The first type, shows lesser and lesser variability as the scale is reduced. Within certain bounds of scales, spatial and temporal variation of a river flow can be interpolated to obtain intermediate values, with a reasonable accuracy. The second type, to which the precipitation belongs to, shows an increase of variability as the scale is reduced. At least in the range of scales that are involved in catchment and urban hydrology, interpolation of precipitation does not yield representative results. A simple example is the temporal variation of rainfall. It is a well established fact that the intensity of precipitation increases with the decreased time step. This intermittency in precipitation demands special techniques in establishing scale relationships.

1.1 Practical problems involving scaling rainfall

Scaling problems can be classified by the dimensional space concerned with: temporal or spatial. This report proposes solutions to some of these problems using theory of multifractals.

The rainfall data acquisition is a time consuming exercise. A large number of years of precipitation data are needed to construct a data set that is climatologically representative of the prevailing conditions of a given locality. Traditionally, rainfall data was recorded in daily basis. One reason for this practice was the limitations of the manual instrumentation that needed human intervention at each record taking. On the other hand, the contemporary hydrological problems and analytical tools did not demand a higher resolution. Consequently, many counties all over the world have at least several good daily rainfall data sets that spans over a century, in many cases.

Recent focus on urban hydrology and natural systems, with drastically decreased response times, are demanding precipitation data with higher temporal resolution. Rainfall data at least of hourly resolution has become a minimal requirement in the modern response studies. However, acquisition of a good set of hourly rainfall data is not a task that can be achieved overnight. One or two years of data have a higher chance of not representing the variability that is important in sound hydrological design. Thus, if daily and hourly rainfall can be related, it allows the water resources engineers to tap into the available resources of daily rainfall to derive hourly rainfall distributions.

The advancements of the regional meteorology and better understanding of global circulation patterns have contributed to improve the weather forecasts at regional scales. Remote monitoring by weather satellites and ground based radar arrays have further enhanced the predictability at regional level.

In order to make use of these climatic predictions for water resources and flood mitigation activities at much smaller scales of operational hydrology, relationships among the regionally averaged precipitation and the local distributions must be established. Even the relationships in a stochastic sense, can contribute much to fields of interests of the hydrological engineer, in areas like long-term flood hazard mapping and water resources availability studies.

[1] Classification of geophysical variables into these two categories become rather subjective, owing to the fact that they always show a mixture of both these properties.

1.2 Fractal as a solution for scaling rainfall

Fractal theory is the recently developed branch of mathematics that concerns with the intermittency and discontinuity of fields. Fractals are objects that show similarity at different scales: the similarity of a part of an object (or a field) to the whole. This similarity has long been observed in natural objects like trees, mountains, river networks, coastal lines and clouds. In the last two decades fractals developed into a distinct field of mathematics that can be used to study the similarity quantitatively. Multifractal is an extension of fractal theory to incorporate fields of measurements.

Physicists have described the scale relationships in numerous types of geophysical fields, including precipitation, using the multifractal theory. The success of the theory in capturing essential features of the 'invarience' of scale – the properties that do not change with scale, has been widely reported in the last decade. Several successful multifractal modes and simulation techniques based on the findings have been developed.

Perhaps the biggest fact in favor of multifractals as means of 'scaling' rainfall is its ability to treat discontinuity and intermittency as an integral part of the core theory, as opposed to many traditional approaches based on continuous fields that consider discontinuities as exceptions. The 'violent' nature of rainfall distribution – the increase of fluctuations as the scale is reduced, can be naturally incorporated into multifractal theory.

1.3 Issues addressed by this dissertation

There were many studies reported hitherto in the literature, concerned with the modeling of precipitation as a multifractal process. A few general purpose multifractal models have been applied to spatial and temporal rainfall data of some regions of the world[2]. However, the gap between these analyses and the application of those findings in operational water resources engineering problems, is still a wide one, requiring further studies to test, model and validate approaches that suit engineering applications. There has been only a very few studies that were targeted at the objective of solving real-world problems.

The primary focus of the work presented in this dissertation was to study new methods that makes it possible to apply the existing knowledge of multifractal theory in solving practical problems of operational hydrology related to scaling of precipitation. While proposing of new multifractal modeling strategies have not been a central issue, several simple methods were developed during the attempts to achieve the above main objective.

There has been no studies about the multifractal scaling properties of Japanese rainfall at the commencement of the work reported here. Investigation of recent literature showed that there were only very few studies involving Asian countries also. Thus, in order to corroborate the scaling behavior in general and validity of multifractal as a model for rainfall in space and time, a number of analyses of Japanese precipitation data have been done.

[2] Most of these studies involved rainfall in Europe, North America and Oceania.

1.4 Organization of the dissertation

Following the formal introduction of fractal as a distinct area of mathematics, two decades ago by Mandelbrot, a large amount of work has been done on the theoretical development of fractals and multifractals as well as on various applications in various fields including geophysics. With special emphasis on modeling on precipitation, a comprehensive review of these findings is done in chapter 2. The area covered in the chapter is too broad to treat each theory and analytical technique completely. Descriptions of the specific models and methodologies that are directly used in the work presented, are supplemented with detailed treatment in each place, where work related to them are presented for the first time. The objective of chapter 2 is to appreciate the landmark developments that lead to the work presented herein and to provide pointers for the search of relevant literature, while the theoretical explanations elsewhere were intended to provide the details that are fairly adequate to repeat the analyses and modeling work presented.

Chapter 3 presents the application of a stochastic rainfall model that was not based on fractal theory, to model rainfall in time. The model, originally intended to be used at a fixed time scale, was applied at a number of different temporal resolutions and the behavior of the model variables at those scales were compared. The patterns revealed by the results of these parameter comparisons suggested that most of the parameters show fractal scaling properties in time. This encouraged the use of fractal theory as a basis for modeling rainfall process, in the main work presented in this dissertation.

A large number of rainfall time series of Japan were analyzed for scaling properties. These results are given in chapter 4. The proposal to model rainfall in time, as a multifractal process is supported by the results. The nature and the extent of the scaling of rainfall in time is examined and the multifractal model parameters that describe that scaling were evaluated. Investigations regarding the dependency of the multifractality on the parameters affecting rainfall, like elevation and geographical location, are also presented.

Chapter 5 presents a modeling approach to derived synthetic hourly rainfall data from daily observations. Scaling of the exceedance probability at different intensity thresholds was used as the basis for relating the scales. All the estimation methods that were available to estimate parameters for multifractal models (which involved exceedance probability or any other measure like statistical moments) required at least several temporal scales available for analysis. However, since the scaling properties of Japanese hourly rainfall data break above 2 days (section 4.2), it was not possible to utilize these methods to derive multifractal properties from daily data – at least in the context of the Japanese rainfall data. In order to overcome this problem, a different method of multifractal evaluation is proposed. Using this method on daily data of quality and length that are typical for operational hydrology, it was possible to evaluate the required multifractal parameters for relating hourly scale to daily scale. A multifractal simulation model was used to generate multifractal fields based on those parameter estimation. Synthetic hourly rainfall series were created from these multifractal fields. The model was validated by comparing the properties of synthetic series with those of the hourly observed rainfall. These comparisons included: comparison of rainfall intensity distributions, time series properties and features that are unique to rainfall, like the quantity and distribution of non-rainy areas and the statistics of individual rain events.

Analysis of hourly rainfall series showed that the rainfall is scaling from 48h to hourly scale. The next natural question is the existence and nature of scaling at scales below 1h. This is also an important investigation, for many hydrological problems related to areas like urban storm drainage design, requires data of temporal resolution of a few minutes. In order to attempt an answer to this question, a high precision rainfall measurement project was conducted in *Ebi* river basin of *Chiba* prefecture. Results of an year of measurement was available at the time of writing. Chapter 6 presents the details of the calibration, field setup and operation of this experiment. The resulting high quality rainfall data was analyzed for multifractal scaling in order to investigate scaling at high resolutions – scales ranging from several hours to several minutes.

For the study of spatial rainfall, two types of data were used. The multifractal analysis of the the first type – rainfall distributions generated by spatially interpolating gauge based observations, are given in chapter 7. Primarily, hourly spatial distributions were analyzed for a period of one year. The seasonality of multifractal properties and the possibility of using multifractals to classify rainfall into types based on distribution patterns, are also discussed. Similar results for daily accumulated spatial rainfall data are given in appendix A. Daily results were compared with a previous report regarding spatial analysis of rainfall in northern Europe. However, there were no similar studies on hourly scale reported.

The other available data source for spatial studies, was the hourly estimations of spatial rainfall by the radar array of the Japan Meteorological Agency[3]. This data has a better coverage and resolution compared with gauge interpolated data, which makes it a useful source of information on the spatial distribution of rainfall. The validity of using the latter source for scaling studies, is examined in chapter 8. A comparison of multifractal model parameters derived from two data sources is presented.

One of the severe shortcoming of using multifractal fields to represent rainfall in space is that multifractals can not treat the spatial heterogeneity that is present in rainfall. There are many reasons including orographic enhancement and slope aspect effects, for heterogeneity in spatial rainfall. While, this heterogeneity is hidden in the (more prominent) random variability at small time integration sizes(e.g. daily scale), it become very much apparent when the integration size is increased to monthly or seasonal level. A rainfall model based on multifractals, but that can take this heterogeneity into account, is proposed in chapter 9. This model treats the spatial variation of rainfall as a combined effect of two independent agents: random variability and the spatial heterogeneity. The model was used to simulate radar based daily spatial rainfall of central part of *Honshu* island and the results are presented.

The results from each chapter are discussed at the end of the chapter. Chapter 10 has a comprehensive discussion of the results presented in the whole dissertation and a conclusions arrived at.

[3]The radar data is calibrated with gauge estimations and are known as radar-AMeDAS.

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