FRACTAL MODELING OF RAINFALL
DOWNSCALING IN TIME AND SPACE FOR HYDROLOGICAL APPLICATIONS

雨のフラクタルモデリング
水文学的応用に向けた時空間ダウンスケーリング

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An Abstract of the Thesis Presented in Partial Fulfillment of the Requirements
for the Degree of Doctor of Engineering
September 2001

There is an urgent need of high-resolution meteorological information to cater for the new focus on the hydrological response at smaller spatial and temporal scales, which has arisen largely due to the problems related to urbanization. Alternative means of estimating precipitation at small scales are beneficial, as high-resolution data acquisition is a time consuming and expensive task. Successful spatial and temporal downscaling methods find many uses in the process of solving practical hydrological problems, due to the fact that they can benefit from the availability of extensive historical records at lower resolutions. These include deriving of high resolution synthetic data, based on low resolution observations; interpolation of the measurements of low density rain gauge networks; combining of data sources of different resolutions and reliability, like gauge based rainfall and weather satellite data; and downscaling of the output of regional climatic models and global circulation models.

In this report, a series of attempts to downscale rainfall fields in space and time using the mathematical theory of fractal scaling is presented. First, a comprehensive review on the recent findings on fractal scaling and its applications on modeling rainfall was done, in order to understand the state of the art of this approach. Last two decades have seen the introduction of fractal and multifractal theories and a considerable amount of theoretical work, including a number of mathematical models on multiscaling. A number of applications on precipitation, most involving rainfall data of various locations of Europe, North America and Oceania, had concluded that rainfall shows fractal scaling properties. There are good reasons for performing this type of analysis in Asia: The nature of the rainfall of Asia is distinctly different from many locations where multifractal studies had been already reported, mainly due to monsoon effect and occurrence of cyclones. The phenomenally high urbanization rate of the region makes it increasingly vulnerable for flash floods. Compared with counties of Europe, North America and Oceania, those of Asia have a severely limited amount of good quality precipitation data at high temporal and spatial intensities. Japan is the special case in Asia, where extensive databases of high-resolution precipitation is available. The investigation of Japanese rainfall is an excellent opportunity to gain an insight into the fractal features of the Asian Rainfall.

The main objective of this study was to apply the already available knowledge of multiscaling of rainfall to device means to solve precipitation related problems in operational hydrology, that involves scaling. However, the questions: if the precipitation is in fact multiscaling and if the multifractal theory can adequately capture the variability in rainfall

in time and space, had to be addressed in order to model the scaling properties accurately. Where possible, the limits of the multifractal scaling regimes were inspected. Most of the analyses and model validations were performed on precipitation data of Japan.

In order to investigate the existence and nature of possible fractal scaling in rainfall, a number of studies were performed. These were the first investigations reported on the multifractal scaling of Japanese rainfall. First, the scaling in time was examined using a large number of gauge observed time-series from all over the Japan. The scaling regime of the investigated data extending from hourly scale to daily scale, found to be breaking at a scale around two days - a value less than that was found in many other studies reported hitherto. The spectral slopes where much closer to unity and were somewhat larger than similar past studies.

The specific problem of deriving hourly synthetic rainfall series from observations made at daily scale was attempted based on the results of multifractal analysis. The apparent break of the scaling regime subsequent to two days made it difficult to use the conventional modeling strategies that require a continuous scaling that spans over at least several different scales. A new modeling approach based primarily on the geometrical properties of characteristic function of multifractal scaling, that can work satisfactorily with the small range of available scales of daily and two-day resolutions, was proposed to overcome the problem. Validation of hourly synthetic rainfall data derived from this method, by statistically comparing that data with hourly-observed series showed that the synthetic data closely resembles observations.

Due to the limitations of resolution and precision of commonly available rainfall data, scaling behavior of rainfall below the hourly scale had not been widely studied. In order to examine the existence and the nature of scaling below hourly scale, several high-resolution rainfall series were obtained by performing a rainfall measuring experiment in several locations in *Maehara*, Chiba prefecture. The results of analysis of data of a period of one year indicated that the scaling properties are extending below hourly scale, at least down to 5min resolution. This result implies that, at least in principal, it is possible to relate a scale as small as 5min to daily rainfall observations.

Spatial scaling of Japanese rainfall data was investigated using gauge interpolated rainfall maps. Before the analysis, the network of rain gauges was examined to evaluate the homogeneity of the distributions and the resolution limits of the analysis, to minimize the effect of the artificial smoothness introduced by interpolation, on the scaling properties derived. Scales between 0.1^0 and 0.8^0 were subjected to analysis. The original time-integration of the rainfall snapshots was one hour. As expected, the analysis indicated a higher variability/intermittency of spatial rainfall than the previously reported studies involving daily accumulations. A secondary analysis with daily accumulations produced multifractal model parameters that are comparable with past studies.

Multifractal properties of spatial rainfall showed some important relationships with the other rainfall related parameters. One is the strong dependency of the former with the large-scale forcing, or the grid-averaged rainfall. The multifractal parameters demonstrated a strong seasonal behavior indicating a sharp anomaly around August, also. This anomaly could be explained by the frequency of occurrence of rainfall of different types in each season.

In order to justify the use of radar-based (but calibrated with gauges) data for scaling studies and modeling, in place of gauge based interpolations, a multifractal model was used to perform a comparative analysis of the two sources. While, the two sources produced closely matching results, there was a discrepancy in scaling properties that could be explained by the differences in extreme values.

The rainfall over a large area generally shows some regular spatial heterogeneity, due to many reasons that include orography and slope aspects. When the scale of temporal accumulation is small, this heterogeneity is hidden in the high variability or the randomness of the process. However, at larger accumulation lengths, rainfall shows distinct patterns that reveal the spatial heterogeneity, for the randomness becomes lesser and lesser with increasing integration volumes. Since multifractals are processes that are statistically homogeneous in space, pure multiscaling fields cannot represent this spatial heterogeneity, which is a inherent characteristic of the aerial precipitation. A new multifractal-based spatial disaggregation model, which can accommodate the spatial patterns at long accumulation sizes, was proposed. The model has two components: A deterministic part that maintains the spatial heterogeneity apparent in large accumulations and a multifractal model to describe the randomness that is predominant in short integration sizes. This separation of rainfall generation into two phases is consistent with the empirical understanding of rainfall over land. The combination of the two components result in a process, which can produce rainfall fields that has both the properties of the small-scale variability and the long-term spatial patterns. Since the heterogeneity and the random variability are independently considered in the model, it has the unique advantage of the ability to use any multifractal model for modeling the latter. This fact was demonstrated by using two multifractal models.

Application of the model on radar based daily rainfall over central Japan, produced encouraging results. The disaggregation based on the large-scale forcing could maintain the observed long-term heterogeneity of the model area accurately. The model was validated by comparing the rainfall intensity distributions at a number of points having small, moderate and large rainfall amounts. The reasons for this model to work well for autumn, summer and spring, but to fail in winter, were explained.

Extensive analytical studies showed that the rainfall in Japan show multiscaling properties. A concrete method to downscale daily rainfall in to hourly scale was proposed and validated. A new spatial rainfall model, which can disaggregate rainfall maintaining the long-term spatial heterogeneity, was implemented. This could create daily rainfall distributions with properties similar to observations. To conclude, this study revealed the potential of the multifractal scaling in solving practical hydrological problem of downscaling rainfall in space and time.